**2.3**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **P** | **Q** | **P**→**Q** | **Q** | **P** |
| T | F | F | F | F |
| T | T | T | T | T |
| F | T | T | T | F |
| F | F | T | F | F |

**12a.**

Row 3 shows that we can have true premises with a false conclusion, which makes the argument invalid.

**12b.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **P** | **Q** | **P**→**Q** | **~P** | **~Q** |
| T | F | F | F | T |
| T | T | T | F | F |
| F | T | T | T | F |
| F | F | T | T | T |

Row 3 again shows true premises with a false conclusion.

**29.**

P → Q

~P

∴ ~Q

Inverse error

**38.**

b. If C is a knight his statement is a contradiction, so C is a knave. Therefore his statement is false and D is a knight.

c. One of E or F is a knave otherwise the statements become contradictory.

**2.4**

**15.**

P

Q

**23.**

P

Q

R

**31.**

(~P ∧ ~Q) ∨ (~P ∧ Q) ∨ (P ∧ ~Q)

≡ [(~P ∧ ~Q) ∨ (~P ∧ Q)] ∨ (P ∧ ~Q) associative law

≡ [(~P ∧ (Q ∨ ~Q)] ∨ (P ∧ ~Q) distributive law

≡ (~P ∧ **t**) ∨ (P ∧ ~Q) commutative and negation laws

≡ ~P ∨ (P ∧ ~Q) identity law

≡ (~P ∨ P) ∧ (~P ∨ ~Q) distributive law

≡ **t** ∧ (~P ∨ ~Q) commutative and negation laws

≡ (~P ∨ ~Q) commutative and identity laws

≡ ~(P ∧ Q) De Morgan’s law

≡ P|Q definition Sheffer Stroke

P

Q

**33a.**

(P ∧ Q) ≡ ~[~(P ∧ Q)] double negative law

≡ ~(P|Q) definition Sheffer Stroke

≡ (P|Q) | (P|Q) example 2.4.7a (~P ≡ P|P)

**33b.**

P ∧ (~Q ∨ R) ≡ (P ∧ ~Q) ∨ (P ∧ R) distributive law

≡ (P ∧ ~Q) ∨ [(P|R) | (P|R)] by 33a result

≡ (P ∧ [Q|Q]) ∨ [(P|R) | (P|R)] example 2.4.7a (~P ≡ P|P)

≡ [(P | [Q|Q]) | (P | [Q|Q])] ∨ [(P|R) | (P|R)] by 33a result

≡ {[(P | [Q|Q]) | (P | [Q|Q])] | [(P | [Q|Q]) | (P | [Q|Q])]} | {[(P|R) | (P|R)] | [(P|R) | (P|R)]}

By example 2.4.7b

**34a.**

~P ≡ P ↓ P

~P ≡ ~(P ∨ P) definition of Pierce Arrow

~P ≡ ~P idempotent law

**34b.**

P ∧ Q ≡ (P ↓ P) ↓ (Q ↓ Q)

P ∧ Q ≡ ~(P ∨ P) ↓ ~(Q ∨ Q) definition of Pierce Arrow

P ∧ Q ≡ ~P ↓ ~Q idempotent law

P ∧ Q ≡ ~(~P ∨ ~Q) defintion of Pierce Arrow

P ∧ Q ≡ P ∧ Q double negative law